Oxford Cambridge and RSA

Thursday 16 May 2019 – Afternoon

AS Level Further Mathematics B (MEI)

Y411/01 Mechanics a

Time allowed: 1 hour 15 minutes

You must have:
- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

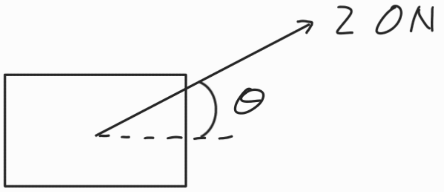
INFORMATION

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

1 A child is pulling a toy block in a straight line along a horizontal floor.
The block is moving with a constant speed of $2\,\mathrm{m\,s^{-1}}$ by means of a constant force of magnitude $20\,\mathrm{N}$ acting at an angle of $\theta°$ above the horizontal.

The work done by the force in $10\,\mathrm{s}$ is $350\,\mathrm{J}$.

Calculate the value of θ. **[3]**



Distance in 2 sec $= 10 \times 2 = 20\,m$

Work done = force × distance travelled in direction of force

$350 =$ horizontal component $\times 20$

$f_h = 17.5\,N$

$\Rightarrow \cos\theta = \dfrac{17.5}{20}$

$\theta = 29.0°$

2 The surface tension of a liquid allows a metal needle to be at rest on the surface of the liquid. The greatest mass m of a needle of length l which can be supported in this way by a liquid of surface tension S is given by the formula

$$m = \frac{2Sl}{g}$$

where g is the acceleration due to gravity.

(a) Determine the dimensions of surface tension. **[3]**

Surface tension also allows liquids to rise up capillary tubes. Molly is experimenting with liquids in capillary tubes and she arrives at the formula $h = \frac{2S}{\rho g r}$, where h is the height to which a liquid of surface tension S rises, ρ is the density of the liquid, and r is the radius of the capillary tube.

(b) Show that the equation for h is dimensionally consistent. **[3]**

In SI units, the surface tension of mercury is $0.475\,\mathrm{kg\,s^{-2}}$ and its density is $13\,500\,\mathrm{kg\,m^{-3}}$.

(c) Find the diameter of a capillary tube in which mercury will rise to a height of 10 cm. **[2]**

In another experiment, Molly finds that when liquid of surface tension S is poured onto a horizontal surface, puddles of depth d are formed. For this experiment she finds that

$$d = kS^{\alpha}\rho^{\beta}g^{\gamma}$$

where k is a dimensionless constant.

(d) Determine the values of α, β and γ. **[4]**

a. $[m] = M$

$[l] = L$

$[g] = LT^{-2}$

Sub in to $m = \frac{2Sl}{g}$:

$$M = \frac{[S]L}{LT^{-2}}$$

$$[S] = \frac{MLT^{-2}}{L}$$

$$[S] = MT^{-2}$$

b. $[S] = MT^{-2}$

$[\rho] = ML^{-3}$

$[g] = LT^{-2}$

$[r] = L$

Sub in to $h = \dfrac{2S}{\rho g r}$:

$$\frac{1}{L^{-3} \times L^{2}} = \frac{1}{L^{-1}} = L$$

$$[h] = \frac{\cancel{M}\cancel{T}^{-2}}{(\cancel{M}L^{-3})(L\cancel{T}^{-2})(L)} = L$$

This is as expected, $\therefore$ equation is consistent

c. $r = \dfrac{2S}{\rho g h} = \dfrac{2 \times 0.475}{13500 \times 9.8 \times 0.1} = 7.18 \times 10^{-5}$ m

diameter $= 1.44 \times 10^{-4}$ m

d. $[d] = L$

$[S] = M T^{-2}$

$[\rho] = M L^{-3}$

$[g] = L T^{-2}$

Sub in to $d = k S^{\alpha} \rho^{\beta} g^{\gamma}$:

$$L = (M T^{-2})^{\alpha} (M L^{-3})^{\beta} (L T^{-2})^{\gamma}$$

Equating powers :

$L: \; 1 = -3\beta + \gamma$

$\gamma = 1 + 3\beta$

$M: \; 0 = \alpha + \beta$

$\alpha = -\beta$

$T: \; 0 = -2\alpha - 2\gamma$

$0 = -2(-\beta) - 2(1+3\beta)$

$0 = 2\beta - 2 - 6\beta$

$4\beta = -2$

$\underline{\beta = -1/2}$

$\Rightarrow \underline{\alpha = 1/2}$

and $\gamma = 1 - 3/2$

$\underline{\gamma = -1/2}$

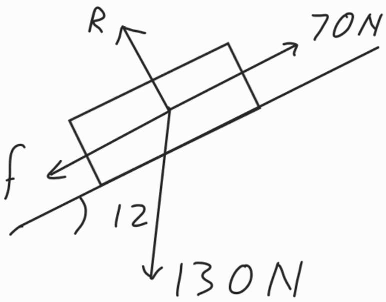
3 A box weighing 130 N is on a rough plane inclined at 12° to the horizontal.
The box is held at rest on the plane by the action of a force of magnitude 70 N acting up the plane in a direction parallel to a line of greatest slope of the plane.
The box is on the point of slipping up the plane.

(a) Find the coefficient of friction between the box and the plane. **[5]**

The force of magnitude 70 N is removed.

(b) Determine whether or not the box remains in equilibrium. **[2]**

a.



Parallel to plane:

$130 \sin 12 + f = 70$

$f = 70 - 130 \sin 12$

$= 42.97\,N$

Perpendicular to plane:

$R = 130 \cos 12$

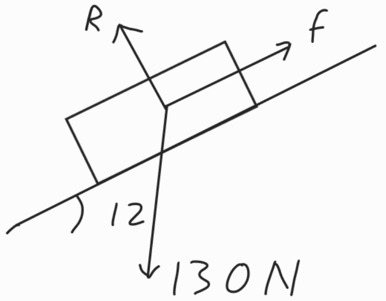
$= 127.2\,N$

$f = \mu R$

$\therefore \mu = \frac{f}{R} = \frac{42.97}{127.2} = 0.338$

$= 0.34$ (2dp)

b.



$$f_{max} = \mu R = 0.338 \times 130 \cos 12$$
$$= 42.98 \text{ N}$$

Force down the plane $= 130 \sin 12 = 27.0 \text{ N}$

$27.0 < 42.98 \therefore$ box remains in equilibrium

4 A shovel consists of a blade and handle, as shown in Fig. 4.1 and Fig. 4.2. The dimensions shown in the figures are in metres.

The blade is modelled as a uniform rectangular lamina ABCD lying in the Oxy plane, where O is the mid-point of AB. The handle is modelled as a thin uniform rod EF. The handle lies in the Oyz plane, and makes an angle α with Oy, where $\sin\alpha = \frac{7}{25}$. The rod and lamina are rigidly attached at E, the mid-point of CD.

The blade of the shovel has mass 1.25 kg and the handle of the shovel has mass 0.5 kg.

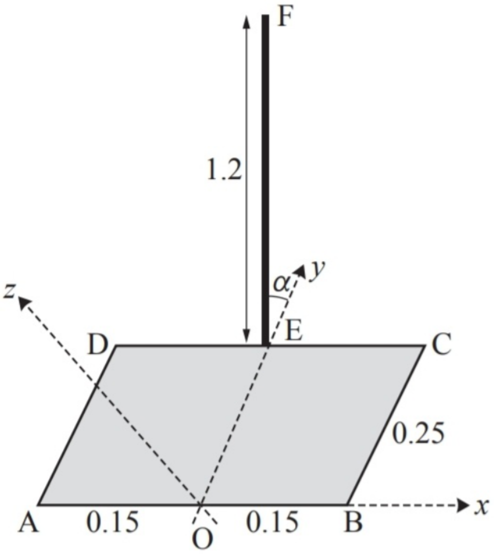


Fig. 4.1

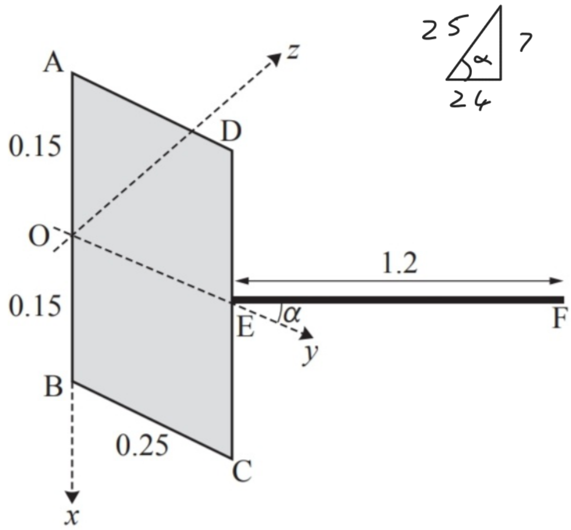


Fig. 4.2

(a) Find,

(i) the y-coordinate of the centre of mass of the shovel, **[5]**

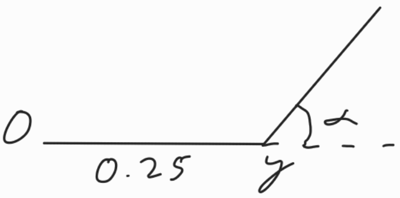
(ii) the z-coordinate of the centre of mass of the shovel. **[2]**

The shovel is freely suspended from O and hangs in equilibrium.

(b) Calculate the angle that OE makes with the vertical. **[2]**

a.i. y-coord for blade $= 0.125$ m

mass of blade $= 1.25$ kg



$\Rightarrow$ y-coord for handle $= (0.25 + 0.6\cos\alpha)$ m

mass of handle $= 0.5$ kg

for whole shovel:

$$m_{total}\,\bar{y} = m_{handle}\,\bar{y}_{handle} + m_{blade}\,\bar{y}_{blade}$$

$$(1.25+0.5)\bar{y} = 0.5(0.25+0.6\cos\alpha) + 1.25\times 0.125$$

$$\bar{y} = \frac{0.125+0.288+0.15625}{1.25+0.5}$$

$$= 0.325$$

a.ii.

z-coord for blade = 0, as the blade is on the xy plane.

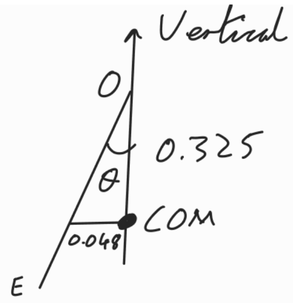
z-coord for handle = $0.6\sin\alpha$

$$m_{total}\,\bar{z} = m_{handle}\,\bar{z}_{handle} + m_{blade}\,\bar{z}_{blade}$$

$$(1.25+0.5)\bar{z} = 0.5\times 0.6\sin\alpha + 1.25\times 0$$

$$\therefore \bar{z} = \frac{0.084}{1.25+0.5} = 0.048$$

b.



$\Rightarrow \tan\theta = \dfrac{0.048}{0.325}$

$\theta = 8.4°$ (2sf)

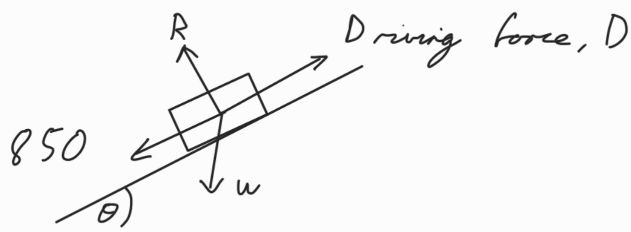
5 A car of mass 4000 kg travels up a line of greatest slope of a straight road inclined at an angle of θ to the horizontal, where $\sin\theta = 0.1$.
The power developed by the car's engine is constant and the resistance to the motion of the car is constant and equal to 850 N. The car passes through a point A on the road with speed $18\,\mathrm{m\,s^{-1}}$ and acceleration $0.75\,\mathrm{m\,s^{-2}}$.

(a) Calculate the power developed by the car. **[5]**

The car later passes through a point B on the road with speed $25\,\mathrm{m\,s^{-1}}$. The car takes 17.8 s to travel from A to B.

(b) Find the distance AB. **[5]**

a.



Σf up the plane $= D - 850 - 4000g \times \sin\theta$

Newton II: $D - 850 - 4000g \times 0.1 = 4000 \times 0.75$

$D = 3000 + 3920 + 850$
$= 7770\ \mathrm{N}$

$P = fv$
$= 7770 \times 18 = 139{,}900\ \mathrm{W}$

$= 140\ \mathrm{kW}$

b. $\Delta KE = \frac{1}{2} m \Delta v^2 = \frac{1}{2} \times 4000 \times (25^2 - 18^2)$

$= +602000 \text{ J}$

PE gained $= mgh = 4000 \times 9.8 \times AB \sin\theta$

$= 3920 AB \text{ J}$

Work done by engine $= Pt = 139860 \times 17.8$

$= 2489500 \text{ J}$

Work done by friction $= fd = 850 AB \text{ J}$

Σ Energy gained $= \Sigma$ work done

$602000 + 3920 AB = 2489500 - 850 AB$

$4770 AB = 1887500$

$AB = 396 \text{ m}$ (3sf)

6 Three particles, A, B and C are in a straight line on a smooth horizontal surface. The particles have masses 5 kg, 3 kg and 1 kg respectively. Particles B and C are at rest. Particle A is projected towards B with a speed of $u\,\mathrm{m\,s^{-1}}$ and collides with B. The coefficient of restitution between A and B is $\frac{1}{3}$.

Particle B subsequently collides with C. The coefficient of restitution between B and C is $\frac{1}{3}$.

(a) Determine whether any further collisions occur. **[7]**

(b) Given that the loss of kinetic energy during the initial collision between A and B is 4.8 J, find the value of u. **[4]**

a. 1st collision:

Let speeds of A and B afterwards $= u_A$ and u_B

Conservation of momentum:

$$5u + 3\times 0 = 5u_A + 3u_B \Rightarrow 5u = 5u_A + 3u_B \quad (1)$$

Newton's law of restitution:

$$\frac{1}{3} = \frac{u_A - u_B}{0 - u} \Rightarrow u_A = u_B - \frac{1}{3}u \quad (2)$$

Sub (2) into (1):

$$5u = 5u_B - \frac{5}{3}u + 3u_B$$

$$8u_B = \frac{20}{3}u$$

$$u_B = \frac{5}{6}u$$

$$\Rightarrow u_A = \frac{5}{6}u - \frac{1}{3}u = \frac{1}{2}u$$

(Continued on next page)

2nd collision:

Let speeds of B and C afterwards = w_B and w_C

COM:

$$3 \times \frac{5}{6}u + 1 \times 0 = 3w_B + 1 \times w_C \quad \Rightarrow \frac{5}{2}u = 3w_B + w_C \quad (1)$$

NLR:

$$\frac{1}{3} = \frac{w_B - w_C}{0 - \frac{5}{6}u} \quad \Rightarrow w_B = w_C - \frac{5}{18}u \quad (2)$$

Sub (2) into (1):

$$\frac{5}{2}u = 3w_C - \frac{5}{6}u + w_C$$

$$4w_C = \frac{10}{3}$$

$$w_C = \frac{5}{6}u$$

$$\Rightarrow w_B = \frac{5}{6}u - \frac{5}{18}u = \frac{5}{9}u$$

$w_B > u_A$ so A and B will not collide again

$w_C > w_B$ so B and C will not collide again

A and C cannot collide without first colliding with B.

$\therefore$ No further collisions occur.

b. Initial KE $= \frac{1}{2} \times 5 \times u^2 = \frac{5}{2} u^2$

$u_A = \frac{1}{2} u$ and $u_B = \frac{5}{6} u$

$\therefore$ final KE $= \frac{1}{2} \times 5 \times \left(\frac{1}{2} u\right)^2 + \frac{1}{2} \times 3 \times \left(\frac{5}{6} u\right)^2$

$= \frac{5}{8} u^2 + \frac{25}{24} u^2 = \frac{5}{3} u^2$

By conservation of energy:

$4.8 = \frac{5}{2} u^2 - \frac{5}{3} u^2$

$u^2 = 4.8 \div \frac{5}{6}$

$u = \sqrt{\frac{144}{25}} = \frac{12}{5} = 2.4 \text{ ms}^{-1}$

7

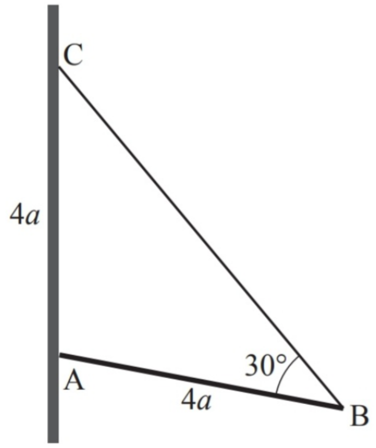
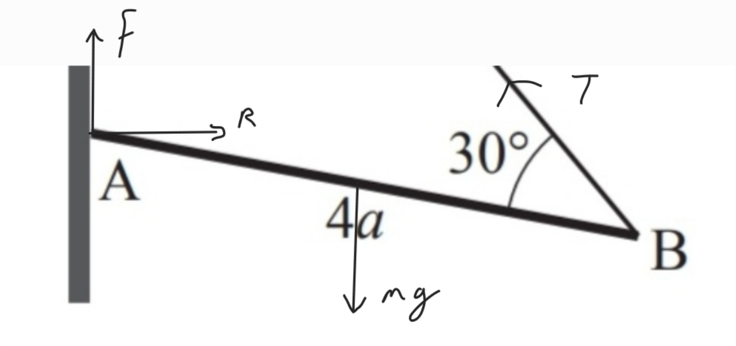


Fig. 7

Fig. 7 shows a uniform rod AB of length $4a$ and mass m.
The end A rests against a rough vertical wall. A light inextensible string is attached to the rod at B and to a point C on the wall vertically above A, where $AC = 4a$. The plane ABC is perpendicular to the wall and the angle ABC is 30°.

The system is in limiting equilibrium.

Find the coefficient of friction between the wall and the rod. **[8]**



Take moments about C:

$R \times 4a = mg \times 2a\cos 30$ (No T or F as they have no component perpendicular to AC)

$R = mg\frac{\sqrt{3}}{4}$

Take moments about B:

$mg \times 2a\cos 30 = R \times 4a\sin 30 + F \times 4a\cos 30$ (sub in $R = mg\frac{\sqrt{3}}{4}$ and rearrange for F)

$F = \left(mga\sqrt{3} - mga\frac{\sqrt{3}}{2}\right) \div 2a\sqrt{3} = \frac{1}{4}mg$

$$f = \mu R$$

$$\mu = \frac{f}{R} = \frac{\frac{1}{4} mg}{mg\frac{\sqrt{3}}{4}} = \frac{\sqrt{3}}{3}$$

Alternative method:

Take moments about A:

$$mg \times 2a \sin 60 = T \times 4a \sin 30$$

$$T = \frac{mga\sqrt{3}}{2a} = mg\frac{\sqrt{3}}{2}$$

Resolve forces horizontally:

$$R = T \sin 30$$

$$R = mg\frac{\sqrt{3}}{2} \times \frac{1}{2} = mg\frac{\sqrt{3}}{4}$$

Resolve forces vertically:

$$f + T\cos 30 = mg$$

$$f = mg - T\frac{\sqrt{3}}{2}$$

$$f = \mu R$$

$$\mu = \frac{f}{R} = \frac{mg - T\frac{\sqrt{3}}{2}}{mg\frac{\sqrt{3}}{4}} = \frac{\cancel{mg} - \cancel{mg}\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}}{\cancel{mg}\frac{\sqrt{3}}{4}}$$

$$= \frac{1 - \frac{3}{4}}{\frac{\sqrt{3}}{4}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$